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The Role of Expectations in Commodity Price Dynamics and the Commodity Demand Elasticity: Evidence from Oil Data

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Discussion Paper in Economics No 16-10

September 2016

ISSN 0143-4543

The Role of Market Expectations in Commodity Price Dynamics and the Commodity Demand Elasticity: Evidence from Oil Data

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September 6, 2016

Abstract This paper examines the contribution of expectations to the oil price dynamics. Classical competitive storage theory states that inventory decision considers both the current and future market condition, and interacts with the spot and expected future spot prices. I model an expectation shock explicitly along with the concurrent supply and demand shocks. This allows for the estimation of the underlying shock processes from the observed price and inventory data and the quantification of their contribution to the price/inventory dynamics respectively. The model is applied to the world crude oil market under assumed price elasticity of demand. The market expectations are estimated to contribute more to the crude oil spot price movements when the demand is assumed to be more inelastic. Thus, the model illustrates the importance of the price elasticity of demand in understanding the price dynamics.

Keywords: crude oil spot price; crude oil supply; crude oil demand; crude oil inventory; expectation shock; dynamic equilibrium model; state space model

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JEL Classification Numbers: Q41, E3.

1 Introduction

Inventory behavior is usually linked to the expectations about future market condition. In the discussion of the causes of the recent crude oil price uprising especially during 2007-2008, one key question is whether speculation played an important role. Regardless of their stand on it, researchers turn to inventory for a better understanding of the speculative or precautionary incentive in the oil market, as anticipation of future increases in oil price could lead to speculative inventory increase and result in immediate price increase[1]. Earlier work like Brennan (1958) already points out the inventory is related to the expected change in price. Applying this intuition in the oil context, Hamilton (2009b) proposes a link between the speculation and the inventory movements. Empirical studies like Kilian and Murphy (2014) argue against a major contribution of speculation where the authors identify the forward-looking element of the real price with data on oil inventories. However, Juvenal and Petrella (2014) find a more important role of expectations also using data on inventories but different macroeconomic indicators.

This model illustrates explicitly the key importance of the price elasticity of demand in interpreting the price dynamics, extending the point made by Hamilton (2009b), Baumeister and Peersman (2013) and Kilian and Murphy (2014), and contributes to the literature on commodity price dynamics, especially the discussion on the role of speculation. By estimating a rational expectations equilibrium model using oil market data while assuming the price elasticity of demand, this paper quantifies the effect of expectations on price movements under different elasticity settings.

[1] The term "expectation" as discussed in this paper will be defined on page 4.

The comparison provides an alternative explanation of the different results in the earlier literature like Kilian and Murphy (2014) and Juvenal and Petrella (2014).

Intuitively, the current inventory and prices are affected by both the current and expected future demand and supply, and their responses are different to changes in the current or expected market condition. Such difference enables identification of the source of change from the price and inventory data. For example, today's strong relative demand to supply will result in higher spot price and lower inventory today. It will also instantaneously result in a higher expected future price due to lower future availability from the depleted inventory (everything else being equal), and an increase in the expected future price that is smaller than that in the spot price. Alternatively, today's expectations of a strong future demand relative to supply will result as well in a higher spot price today, due to the lower current availability from the accumulating inventory in response to such expectations, but the increase in the expected future price would be larger than that in the spot price in this case. The estimation of the model uncovers the underlying stochastic processes driving the observed prices and inventory data, and thus the role of expectations.

This paper is the first to quantify the effect of expectations using a structural model. While earlier empirical work like Kilian and Murphy (2014) and Juvenal and Petrella (2014) adopt similar intuition in identifying the expectations, one advantage of the structural framework to earlier work is the precise mapping of mathematical expression to economic interpretation.

The structural framework allows for not only analyzing the role of expectations in the price inventory dynamics with explicitly defined price elasticity of demand, but also mathematically defining the expectations.

It's revealed that the price elasticity of demand is a key parameter determining the magnitude of the price and inventory responses to shocks. The more inelastic the demand, the larger the magnitude of the prices and inventory responses to changes in the market condition. Given the observed data and its variances, when the assumed elasticity is different, the model has to assign the contribution of the expectations differently in order to reconcile with the observed reality. Depending on the demand data used, the different implied demand elasticity might result in different results on the contribution of the expectations.

It's also worthwhile discussing briefly what the "expectation" in the model captures. The "expectation" in the model specifically refers to the innovations and macroeconomic activities that could affect the commodity market supply and demand with a delay, in the style of the news shock that has been discussed by Beaudry and Portier (2006) and adopted by a large macroeconomic (DSGE) literature like Davis (2007), Barsky and Sims (2011), Jaimovich and Rebelo (2009) and others.

More specifically, the "expectation" process in the model has no contemporaneous but only lagged effect on the supply and demand. The idea is that agents in the market may learn about the production capacity that has been recently installed and will be implemented in the future, at which time they expect the supply to rise. Similarly, agents could learn that commodity will be utilized with higher efficiency in the future production at which time they expect the demand to shift. Such expectations have no effect on the current market supply and demand condition, but do affect agents' current inventory decision, and affect the spot and expected future prices. It's such expectations that are referred to as the "expectation" in the model.

Despite the different modelling strategies, the results of the paper is comparable to the earlier literature. The expectation as defined in this structural model overall shares similarities with and is comparable to that in earlier literature like Juvenal and Petrella (2014), Kilian (2009), Kilian and Murphy (2014), Baumeister and Peersman (2013) and others. VAR can be interpreted as the reduced form of structural model, and the sign and boundary constraints adopted to identify expectations (or "precautionary demand" as referred to) in earlier work are comparable to the impulse response functions to the expectations in this structural model.

However, this paper differs from earlier work in the assumption on the oil supply. This model views the supply as exogenous, not affected contemporaneously by the demand side (global demand, precautionary, and speculative motives). The observed changes in the world oil production are largely driven by the aggregated natural variation of the field production, and newly-started fields which have been planned several years in advance, rather than by the concurrent demand-induced price changes[2]. This view does result in different identification constraints of the underlying shocks, which will be discussed in detail in Section 2.

This model also differs from one strand of earlier storage and price dynamics literature like Wright and Williams (1982, 1984) and Deaton and Laroque (1992, 1995, 1996) and the more recent Dvir and Rogoff (2010) and Arseneau and Leduc (2013) in modelling inventory stock-out. Instead, observing that oil market doesn't typi-

[2]Recent works using field-level production data from North Sea and Texas (Hurn and Wright (1994), Mauritzen (2014) and Anderson et al. (2014)) provide strong evidence that the impact effect of the current price (level and volatility) on production is negligible. In other words, the short-run supply curve of individual fields is almost vertical.

cally experience stock-outs, I model a non-linear marginal convenience yield function as in Pindyck (1994) such that when the inventory approaches zero, the marginal convenience yield approaches infinity. Intuitively this setting implies that it's always beneficial to hold inventory. As a result the inventory will always stay positive[3].

The paper is planned as follows. Section 2 introduces the model. Section 3 discusses the theoretical implications on the price-inventory dynamics in an equilibrium model under rational expectations. Section 4 presents the estimation results and the discussion of the role of the shocks during the past price movements. It compares two cases of price elasticities of demand to illustrate the difference this key parameter makes in the interpretation of price dynamics. Section 5 concludes.

2 The Model

This section sets up the model for oil market equilibrium with inventory. Although it has been interpreted in the oil market context, the model can be generally applied to most storable commodity markets in which no stock-out has been observed. In this model of the world oil market, the price is determined by the world supply, the demand for consumption and the overall economic performance. The quantities supplied and demanded are not necessarily the same, as the market also has the demand for inventory, based on the current market and the expectations of the future.

[3]Similarly, Eichenbaum (1984) also argues for the technological in addition to the speculative reason for voluntarily-held inventory.

2.1 Demand for Oil

First, I model the demand side of the world oil market. I start with a general inverse demand function for crude oil, where the oil price P_t is determined by the oil consumption Q_t^d, and a measure of overall economic performance Y_t^d. In specific, Y_t^d captures the shifts of the demand curve driven by the global economic fluctuations. For example, Kilian (2009) has argued that the demand for industrial raw materials has been fuelled by the emerging economies in Asia such as China and India after 2002. For now, let Y_t^d denote a measure of overall economic performance, which can be some function of world GDP, or industrial output, or the index of world economic activities as proposed by Kilian (2009). The inverse demand function of oil:

$$P_t = P(Q_t^d, Y_t^d) \tag{1}$$

is decreasing in Q_t^d and increasing in Y_t^d. I further posit this inverse demand function to be homogeneous of degree zero, i.e. only the consumption relative to the overall economic performance matters, as oil consumption and world economic performance is highly correlated. Thus I can use a CES inverse demand function:

$$P_t = c(\frac{Q_t^d}{Y_t^d})^{-\frac{1}{\gamma}} \tag{2}$$

where c is a scalar and γ measures the price elasticity of demand. Denote the available inventory at the beginning of period t by N_t, and the inventory held for next period $t+1$ by N_{t+1}. In the market equilibrium, the crude oil consumption Q_t^d equals to the crude oil production Q_t^s less the change in inventory $N_{t+1} - N_t$:

$$P_t = c(\frac{N_t + Q_t^s - N_{t+1}}{Y_t^d})^{-\frac{1}{\gamma}} \tag{3}$$

2.2 Inventory Decision

In addition, the need for inventory-holding arises from the uncertainty about the future. A profit-maximizing oil producer (or buyer) in a competitive market makes decision with regards to its inventory-holding following the first-order condition when the inventory is positive[4]:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \qquad \text{if} \quad N_{t+1} > 0 \tag{4}$$

where MIC is the net marginal cost of holding inventory, which includes the physical cost of storage as well as the convenience of storage (see Brennan (1958) and others). Whenever positive inventory is held, an optimal inventory decision N_{t+1} at time t would be such that the resulting net marginal cost of holding inventory $E_t[MIC_{t+1}]$ would be just covered by the marginal revenue, or the expected intertemporal price change $\beta E_t[P_{t+1}] - P_t$.

Since in the commodity market, zero inventory is rarely observed, the net marginal cost of holding inventory is modeled such that N_{t+1} would always be positive. Namely I assume that the net marginal cost converges to negative infinity when inventory is drawn down to near zero. Thus, even when the price is expected

[4]This first-order condition is the same regardless of whether it's the producer or the buyer holding the inventory

to fall and the expected intertemporal price change $\beta E_t[P_{t+1}] - P_t$ is very negative, the inventory still won't be drawn out completely. Intuitively, inventory facilitates production and delivery scheduling and avoids stockouts in the face of fluctuating demand and changing supply technology. These benefits motivate producers to hold inventory even if they expect the price to fall, as discussed in Brennan (1958). I follow the exponential function for the net marginal cost of holding inventory as suggested by Pindyck (1994)[5], assuming that there is a constant marginal inventory-holding cost δ, and that the net marginal cost is affected positively by the current price as well as the inventory held relative to the quantity demanded. I further introduce an inventory adjustment cost, following earlier literature like Eichenbaum (1984), observing that the relative inventory (the inventory held relative to the quantity demanded) data is much less volatile compared to the price even after removing the seasonality.

$$MIC_{t+1} = P_t * [\delta + \alpha(\frac{N_{t+1}}{N_{t+1} + Y^s_{t+1} - N_{t+2}})^{-\phi} + \Theta(\frac{N_{t+1}}{N_t}) - \beta * \Theta(\frac{N_{t+2}}{N_{t+1}})] \qquad (5)$$

The net marginal cost of storage here takes into consideration the physical cost of holding inventory δ, the intangible benefit of inventory-holding to avoid stock-out (the exponential part; $\alpha < 0$) and the inventory adjustment costs Θ (which is a function of relative inventory changes) for both current and next periods. The exponential part captures the intangible benefit of inventory-holding in a way such that the benefit would be low when the inventory level is already high relative to

[5] Pindyck (1994) refers to the negative net marginal cost of storage as "the net marginal convenience yield", and proposes an exponential form for the latter based on the observation that the scatter plot of relative inventory against the net marginal cost of storage is nonlinear.

demand, and vice versa. Such setting guarantees that inventory level is never drawn down to zero. Θ is assumed to be zero when there's no change in inventory, and to have constant marginal adjustment cost (Θ'). More detailed discussion of the parameters and the functions will be available in later section of the model solution and its estimation.

2.3 Exogenous Shocks in the Model: Modelling Expectation

The key part of the model is modeling the factors driving the price and inventory, including demand, supply and expectations. The model itself doesn't attempt to explain how demand, supply and the expectations about them arise, and thus treat them as exogenous. Like the identification restrictions of the reduced form analysis, assumptions on these exogenous factors are important for the model's economic intuition and the solution. This section will present how the exogenous processes of supply, demand and expectations are modeled.

On the supply side of the market, the world crude oil supply plotted in Figure 1 appears to contain a stochastic trend and the log first-difference appears to be stationary. The log of world crude oil supply can be reasonably assumed to follow a random walk process with a drift.

$$\log(Q_t^s) = \log(Q_{t-1}^s) + \log(\mu_t^s) \tag{6}$$

$$\log(\mu_t^s) = \bar{\mu} + \epsilon_t^{\mu} \sim N(0, \sigma_\mu^2) \tag{7}$$

The process for the demand side is modeled implicitly. The demand shifter, or the process for overall economic activities Y_t^d can be thought of as some function of either world GDP or industrial index as discussed earlier. Regardless which one of these measures best approximates Y_t^d, the process is quite possibly non-stationary. However, in the oil/commodity market context, it's also reasonable to think that the overall economic activities are overall balanced with the supply in the long run, as strong economic activities encourage new production capacity instalment and new exploration, and weak economic activities lead to fewer drilling activities. Thus, instead of modeling Y_t^d explicitly as another random walk, I model the stationary relative supply, $\frac{Q_t^s}{Y_t^d}$:

$$\log \frac{Q_t^s}{Y_t^d} = y_t^\tau + y_t^c \tag{8}$$

$$y_t^\tau = \rho^\tau y_{t-1}^\tau + n_{t-1}^\tau + \epsilon_t^{y_\tau} \qquad \epsilon_t^{y_\tau} \sim N(0, \sigma_{y_\tau}^2) \tag{9}$$

$$y_t^c = \rho^c y_{t-1}^c + \epsilon_t^{y_c} \qquad \epsilon_t^{y_c} \sim N(0, \sigma_{y_c}^2) \tag{10}$$

$$n_t^\tau = \rho^{n_\tau} n_{t-1}^\tau + \epsilon_t^{n_\tau} \qquad \epsilon_t^{n_\tau} \sim N(0, \sigma_{n_\tau}^2) \tag{11}$$

This stationary assumption on $\frac{Q_t^s}{Y_t^d}$ is especially important for solving the model (this will be discussed in next subsection).

It's worth noting that the above assumptions view the supply as exogenous to the demand while the two remain cointegrated. The view sharply contracts with the identification restrictions of Kilian (2009), Kilian and Murphy (2014), Juvenal and Petrella (2014) and others. The assumption that the supply shock ϵ_t^μ is independent of the shocks ($\epsilon_t^{y_\tau}$, $\epsilon_t^{y_c}$ and $\epsilon_t^{n_\tau}$) to the cointegration relationship ($\log \frac{Q_t^s}{Y_t^d}$) implies that the supply is not affect by the demand side. This is in lie with the empirical findings that the demand side shocks don't not affect the supply (see Hurn and Wright (1994), Mauritzen (2014) and Anderson et al. (2014)).

Namely, the relative supply process is assumed to contain a persistent part y_t^τ, a temporary part y_t^c and an expectation part. The persistent and temporary parts are both AR(1) processes, with $\rho^\tau > \rho^c$. The expectation n_t^τ is modeled as an AR(1) process with autoregression coefficient ρ^{n_τ}.

The expectation n_t^τ is modeled similarly to the news in the DSGE literature. It captures the events that could affect the market demand and supply with delay as Equation 9 shows. When the market expectations at t changes, even though the relative supply in the current period t isn't affected, rational market participants would still respond right away to the expectation change by adjusting inventory which results in contemporaneous price change. This expectation in the model captures the forward-looking component of price determination in the market: if the market agents believe that the price would be higher in the future, such expectations would drive up the price and inventory today.

2.4 The Model Overview and Its Equilibrium

Normalization of some variables is necessary in order to solve for the steady state of the model and the equilibrium path since they contain trends (Q_t^s, Y_t^d). I follow the macroeconomic literature in treating the variables with a trend, and normalize them by the world supply.

Such normalization of variables in Equation 3 results in the "relative supply" $\frac{Q_t^s}{Y_t^d}$, which I will denote by lower letter, $q_t^s = \frac{Q_t^s}{Y_t^d}$. It is assumed to be stationary (see Equation 8 to 11) so that the model has a steady state. The inventory is also normalized, resulting in $n_{t+1} = \frac{N_{t+1}}{Q_t^s}$. The normalized inventory variable n_{t+1} can be thought of as the "effective inventory" level.

Equation 3 then can be rewritten in terms of the "effective inventory" n and the "relative supply" q:

$$P_t = c[(n_t/\mu_t^s + 1 - n_{t+1}) * q_t^s]^{-\frac{1}{\gamma}} \tag{12}$$

Similarly, the normalization of variables in Equation 4 and 5 rewrites the equations as:

$$P_t = \beta E_t[P_{t+1}] - E_t[MIC_{t+1}] \tag{13}$$

$$MIC_{t+1} = P_t * [\alpha(\frac{n_{t+1}/\mu_{t+1}^s}{n_{t+1}/\mu_{t+1}^s + 1 - n_{t+2}})^{-\phi} + \delta + \Theta(\frac{n_{t+1}}{n_t/\mu_t^s}) - \beta * \Theta(\frac{n_{t+2}}{n_{t+1}/\mu_{t+1}^s})] \tag{14}$$

where $\mu_{t+1}^s = \frac{Q_{t+1}^s}{Q_t^s}$, as defined in Equation 6.[6]

Now the full model is written in the normalised terms as Equations 12, 13 and 14, along with the exogenous processes μ_t^s, y_t^τ, y_t^c and n_t^τ given by equations 7 8 9 10 11.

The equilibrium path is defined as follows: taking as given the exogenous processes μ_t^s, y_t^τ, y_t^c, n_t^τ and the resulting q_t^s, and an initial stock of effective inventory n_0, the equilibrium of the model is a sequence of $\{P_t, n_{t+1}\}$ that satisfies: the optimality conditions of inventory-holding 13 and 14; the market clearing condition 12.

[6] Note that $log(\mu_{t+1}^s)$ is the world supply growth rate.

3 Solving the Model

In this section, the model is solved and "price policy" and "storage policy" can be obtained which relate the equilibrium price and inventory decision to the current and the expected market demand/supply. The solution will be illustrated using the impulse response functions of the price and inventory to the underlying shocks. The impulse responses will also be compared to the sign restrictions widely adopted in recent empirical literature as well.

More importantly, the solution of the model reveals that the price elasticity of demand (γ) plays a key role in the magnitude of the price and inventory responses. The impulse response functions show that everything else being equal, the more inelastic the demand, the larger the magnitude of the price and inventory responses to the underlying shocks, especially to the expectation shock.

3.1 Model Solution

Specifically, for arbitrarily-set parameters, I log-linearize the model around its deterministic steady state and solve the resulting linear rational expectations model as in Blanchard and Kahn (1980). The resulting linearized model links the equilibrium price P_t and the effective inventory n_{t+1} with underlying relative supply processes (in terms of their deviations from the steady state values). Current-period spot price (P_t) and next-period effective inventory (n_{t+1}) are determined based on the predetermined current-period effective inventory (n_t) and the realized shocks ($\hat{\mu}_t^s$, y_t^τ, y_t^c, n_t^τ). This solved model can be written in a state space form with the currently available effective inventory (n_t) and the exogenous shocks ($\hat{\mu}_t^s$, y_t^τ, y_t^c, n_t^τ) as the state variables, and the spot price (P_t) as the observed variable. The expected spot price ($E_t(P_{t+1})$) could also be attained. Appendix A offers more details on the solution algorithm.

3.2 Simulated Impulse Response Functions

In this section I arbitrarily set the parameters and present the impulse response functions of price and effective inventory to different shocks. The baseline parameterization is summarized in Table 1.

, and then the resulting impulse response functions to the three shocks to the relative supply under the arbitrary parameterization in Figure 3. More importantly, the magnitude of the impulse responses greatly depends on the price elasticity of demand. Figure 4 compares the impulse response functions under different price elasticity of demand (γ). This is the key to the estimation results discussion later.

3.2.1 Why the "Expectation Shock" is Named So

I first show how the different shocks effect the world relative supply in Figure 2. Figure 2 plots the impulse response functions of the relative supply to one-standard deviation shocks. All shocks have been normalized to cause an increase in the relative supply. Both the persistent and temporary shocks cause a peak increase immediately in the relative supply, while the expectation shock causes zero change in the initial period. Instead, the peak effect takes place after several periods[7]. This striking

[7]The exact peak time and the magnitude of the peak effect of expectation shock depends on the specific parameterization of the stochastic process, thus Figure 2 is only for qualitative illustration

difference illustrates the intuition discussed earlier: a shock to the expectation represents some event that is known to affect the relative supply with a delay. Such event doesn't cause any change in the current relative supply; instead, it's only after the event being known by the market participants that the relative supply is affected.

The effect on the world relative supply translates immediately into changes in the equilibrium price and inventory. Figure 3 plots the impulses response functions of the spot price P_t, expected change in price $E(P_{t+1} - P_t)$ and the effective inventory n_{t+1} following different one-standard deviation shocks. All shocks have been normalized to cause an initial increase in the real spot price of oil. Again, overall the impulse response functions to persistent and temporary shocks are similar. They both cause an initial real price increase that gradually dissipates, accompanying with a decrease of smaller magnitude in the expected change in price ($E(P_{t+1} - P_t)$) and a decrease in the effective inventory. While all the shocks are of the same size (see Table 1 for the parameters setting), the price response to the persistent shock is of a larger scale compared to that to the temporary shock, while the inventory response to the persistent shock is of a smaller scale relatively.

The price and inventory responses to an expectation shock are also immediate, despite the fact that the relative supply is not changed at all in the initial period (see Figure 2 where the contemporaneous response of relative supply to the expectation shock is zero in the first period). The intuition is, knowing of some event which will cause a future shortage of supply relative to demand, market participants start to accumulate the inventory right away. This mechanism captures how "expectations"

in these aspects.

work thus the name. The expectation-driven demand for inventory effectively lowers the amount of oil currently available for consumption and drives up the spot price immediately, even though the current supply and consumption demand for oil remains the same. Since the peak effect on the relative supply takes place after several periods (see Figure 2), spot price keeps picking up after the initially increase. The expectated future price increases more than the spot price, as implied by the positive expected change in price$E(P_{t+1} - P_t)$, while the effective inventory accumulates.

3.2.2 How the Identification of the Shocks compares to the Literature

The above responses to the expectation shock are consistent with what the literature identifies as forward-looking behavior. Kilian and Murphy (2014) and Juvenal and Petrella (2014) adopt similar arguments in constructing their VAR models of crude oil market. The sign restrictions adopted to identify "speculative demand shock" in Kilian and Murphy (2014) or "other demand shock" in Juvenal and Petrella (2014) posit that, the shock have positive impact effect on inventory accompanying a spot price increase, similar to the impulse responses of the spot price and effective inventory discussed above. The different price-inventory dynamics in response to different shocks will enable us to uncover them.

This model also extends the economic intuition adopted by VAR identification. Figure 3 shows that the more persisting shocks appear to affect price more and inventory less relative to more temporary shocks, other things being equal. Intuitively, when the market expects the disruption to relative supply to last long, there would be relatively less incentive to drawn down inventory by a large amount immediately.

As a result, the price response would be expected to be larger for an extended period, with lesser intertemporal change in the inventory after a more persistent shock.

3.2.3 How Important the Price Elasticity of Demand is

Figure 4 illustrates that, other things being equal, the more inelastic the demand is, the larger the magnitude of the inventory and price responses to the underlying shocks, especially to the expectation shock[8]. While the larger magnitude of the price response under less elastic demand is straightforward to understand, the larger magnitude of the inventory response needs more discussion. Take the impulse response function to the temporary shock y_c^τ for example. A negative temporary shock (stronger demand relative to supply) will result in immediate increase in the spot price (P_t) and withdrawal of the inventory (n_{t+1}). Suppose the magnitude of the inventory response remains the same regardless of the price elasticity. This implies the oil availability remains the same for the next period. However, with a lower price elasticity of demand the current price (P_t) increase is larger, so is the expected spot price ($E(P_{t+1})$). Overall the relative increase of the spot price compared to the expected future price ($P_t - E_t(P_{t+1})$) is larger with a lower elasticity. This implies more costly inventory holding (see Equation 4); in other words, the inventory is too high with the assumed inventory withdrawal. Thus the inventory (n_{t+1}) has to be drawn down more to bring the market back into equilibrium.

To summarize, the structural model makes use of the additional information of the magnitude in the estimation. In the next section, the model is brought to data

[8] Aside from γ, the three cases in Figure 4 all have the same parameters setting as listed in Table 1

and the shocks behind the oil price fluctuations are estimated.

4 Estimation Results

In this section I present the data and the model estimation. The estimation results include the parameter estimates, the estimated impulse response functions, the estimated underlying shocks and their contribution to the price and inventory dynamics. As will be shown, the price elasticity of demand γ plays an important role in the estimation of the shocks' contribution.

4.1 Data and Estimation

4.1.1 Data

The model is estimated using monthly data from 1987 January to 2014 November. The estimation uses the real spot and futures (1-month) prices, the effective inventory and the world crude oil supply growth rate.

An overview of the data is presented in Figure 5[9]. For the prices (P_t and $E_t P_{t+1}$) I use real spot and futures (1-month) prices of WTI deflated by monthly US CPI (1982-84=100)[10].

For the effective inventory n_{t+1}, I use the ratio of the world inventory and the world supply as discussed earlier in the model solution. While the world inventory of crude oil is not available, I use OECD inventory as its proxy, which is end-of-month

[9] The data has been demeaned for the estimation.

[10] To use 1-month WTI futures price for $E_t P_{t+1}$ in the model assumes that there's no risk premium in the 1-month futures price. Given the short maturity length, this assumption is not that unreasonable.

US commercial inventory of crude oil scaled by the ratio of OECD to US petroleum products stock, following Hamilton (2009a), Kilian and Murphy (2014) and Juvenal and Petrella (2014). I also adjust the seasonality in the effective inventory data by including additional monthly dummies in the state equation (see Appendix A.2).

For the world crude oil supply growth rate $\log(\mu_t^s)$, I use the log first-difference of the world supply, which is available from Energy Information Administration (EIA).

4.1.2 What Parameters are Estimated and Why γ is Arbitrarily Set

The parameters estimated are listed in Table 2 and 3. The solved linearized model allows for estimation of the parameters for the shock processes (ρ's and σ's), the parameters in the net marginal cost of inventory holding (δ and Θ' in Equation 14[11]) and the monthly dummies for the effective inventory.

Two scalars, α in the net marginal inventory cost function, and c in the world demand for oil, are calibrated from the steady state condition using the estimated parameters and the data. This is because α and c only matter to the levels of the variables, not their deviations from the steady state. Once the model is linearized around the steady state and the variables are written in terms of their deviations from the steady state, α and c no longer appear in the solved model and don't matter to the dynamics of the deviations. As result, they cannot be estimated using the logged differenced data presented in Figure 5. (Appendix A presents the log-linearized model and shows that it no longer contains α and c as the parameters).

[11]Appendix A shows that the log-linearized model no longer contains Θ but only its first derivative Θ' evaluated at the steady state, which is assumed to be a constant (see discussion in 2.2). Similarly, ϕ and δ always appear together as $\phi(1-\beta+\delta)$ and cannot be identified separately. Thus, ϕ is arbitrarily set as estimated by Pindyck (1994) and only δ is estimated.

Two key parameters, γ, the short-run price elasticity of demand for crude oil, and β, the monthly depreciation rate, have to be arbitrarily set as they cannot be estimated without any data on the demand side. However, the demand elasticity is potentially important for the estimation as discussed earlier. Thus the range in the literature on demand elasticity estimation is used as a reference: 0.05 to 0.44 (Dahl (1993), Cooper (2003), Baumeister and Peersman (2013), Bodenstein and Guerrieri (2011), Kilian and Murphy (2014))[12] with admissible values as low as 0.01 (see Baumeister and Peersman (2013)). I pick the literature average 0.25 and a lower-bound 0.02 for possible γ's in the estimation. The monthly depreciation rate is set to be 0.997.

4.2 Estimated Parameters and Impulse Response Functions

In this subsection I present the estimation results under different demand elasticity settings ($\sigma = 0.25$ and 0.02). I also discuss how the estimated results especially the impulse response functions relate to the existing literature.

4.2.1 Estimated Parameters

Tables 2 and 3 summarize the estimation results under different demand elasticity settings[13]. In Table 2, for both cases ($\sigma = 0.25$ and 0.02) all parameter estimates are significant at 99% confidence level. In Table 3, estimates of the monthly dummies indicate that effective inventory tend to be higher during colder months than warmer

[12]See Hamilton (2009a) for a summary of the estimates in the literature in Table 1. Kilian and Murphy (2014) also provides a brief survey of the estimates.

[13]The model is estimated by maximum likelihood and various initial guesses of the parameters have been tried. The estimation results presented here have the highest likelihood.

months (dummies for colder months tend to be negative)[14]. However, the dummies estimates are significant only for the case of $\sigma = 0.25$, even though the point estimates for both cases are similar.

More importantly, for the case of $\sigma = 0.25$, two shocks (the persistent and expectation shocks) are identified as random-walk ($\rho^{\tau} = 0.9993$, $\rho^{n_\tau} = 0.9991$) and the temporary shock is stationary ($\rho^{c} = 0.0451$); for the case of $\sigma = 0.02$, only the persistent shock is random-walk ($\rho^{\tau} = 0.9998$), the temporary shock is stationary ($\rho^{c} = 0.0279$) and the expectation shock is white-noise ($\rho^{n_\tau} = 0.0000$). Also, the shock sizes are larger for the case of $\sigma = 0.25$, where the highest standard deviation is $\sigma_{y_\tau} = 0.0197$, compared to the case of $\sigma = 0.02$, where the highest is $\sigma_{n_\tau} = 0.0088$.

4.2.2 Estimated Impulse Response Functions

Figure 6 shows the the estimated responses of the relative supply to shocks under different γ settings. Again all shocks are one-standard deviations, normalized to cause an increase in the relative oil supply. The different settings of γ result in different estimated shock dynamics. The different peak effect sizes and timing reflect both different shock volatilities and persistences. As discussed earlier, lower demand elasticity σ works as a magnifier of the price and inventory responses. When γ is small and the demand is inelastic, the observed volatility in the price and inventory data is hard to reconcile with the shocks with large volatility and high persistence. As a result, the estimated shocks tend to have smaller standard deviation and lower persistence for the case of $\sigma = 0.02$.

[14] Similarly, Byun (2012) finds a higher utilization of inventory in refining production for warmer seasons.

The different estimated shock dynamics is more apparent when comparing the impulse response functions of the price and inventory. Figure 7 plots the impulse response functions of the price and inventory under different γ settings. All shocks are one-standard deviations, normalized to cause an increase in the real spot price of oil. Both sets of impulse response functions overall show the same direction of changes[15], and are consistent with the sign restrictions adopted in the reduced-form models[16].

However, the two sets of dynamics over time are very different. Most prominently, the expectation shock has much larger effect on the spot price in the case of $\gamma = 0.02$. Meanwhile, in general the persistent and the expectation shocks have smaller effect on the inventory and the expected change in price ($E(P_{t+1} - P_t)$) for the lower γ case.

4.2.3 The Results and the Literature

Hamilton (2009b) argues that in presence of high price, different changes in inventory would help identify different type of shocks behind. Consistent with this argument, the model shows that when the spot price is positively affected, the persistent and temporary shocks cause negative changes in the effective inventory while the expectation shock causes negative ones (Figure 7). Furthermore, when the demand is more elastic ($\gamma = 0.25$), the difference in the inventory responses to the

[15] For example, in both cases, the persistent shock causes positive changes in the spot price.

[16] The impulse responses to the expectation shocks are consistent with the sign restrictions of the "speculative demand shock" in Kilian and Murphy (2014) and "other demand shock" in Juvenal and Petrella (2014). The impulse responses to the persistent and temporary shocks do not exactly match the sign restrictions though as here this model doesn't differentiate the demand and supply shocks but rather takes them as a composite.

concurrent and the expectation shocks is larger.

Kilian and Murphy (2014) postulate that a positive speculative demand shock is "associated with an immediate jump in the real price of oil". Consistent with this argument, the model shows that the expectation shock has impact effect on the price (Figure 7) without affecting the current market supply and demand for consumption (6). Furthermore, when the demand is more elastic ($\gamma = 0.25$), the impact effect is smaller.

Hamilton (2009b) also discusses the extreme case where speculation drives up spot price without a change in inventory. In this model, the case of $\gamma = 0.02$ might be considered as the extreme case, where the expectation shock results in a positive response of the price and very small positive impact effect on the inventory. Similarly, Parsons (2010) argues that expectations of higher future price doesn't necessarily lead to inventory accumulation if the entire term-structure is elevated due to speculative incentives. In the case of $\gamma = 0.02$, the flat response of the expected change in price ($E(P_{t+1} - P_t)$ and the impact positive response of the spot price after the expectation shock would correspond to an "elevated futures curve", and the positive inventory response is small.

4.3 Estimated Cumulative Effects of the Shocks

The different settings of γ result in different estimated shock dynamics, and ultimately result in different decomposition of the price and inventory behavior. Along with the estimated parameters, the state variables of the state space model, or the shocks, are also estimated. This allows for computing the cumulative effect of each shock on the real prices of oil and the effective inventory, to understand the historical price evolution.

It's worth noting that in this model the state variables include both the effective inventory and the exogenous shocks. As a result, to separate out the effect of a certain exogenous shock from that of the initial effective inventory and other shocks, the cumulative effect of a shock is calculated as the hypothetical price and inventory series given the Kalman-smoothed time series of the shock of interest, keeping the initial effective inventory and all other shocks as zeros. More details are in Appendix B.

Overall the estimation results match the general understanding of the market. In some cases, the results even match the specific date of historical events. Figure 8 plots the decomposed contribution of each shock on the observed real spot and futures prices and the effective inventory when $\gamma = 0.25$. Overall, under the assumption of $\gamma = 0.25$, the model estimates a persisting tighter market after 2000 as indicated by the cumulative effect of the persistent shock: the persistent shock contributes to most of the price increase after 2000, except for a short period during 2008-2009 and towards the very end of the sample period (November 2014); it also contributes to the continued withdrawal of the effective inventory, especially in 2000-2008. The model also estimates an expectation of tight market condition at the beginning of the sample period, and after January 2005: the expectation shock contributes to the price increase at the beginning of the sample period (from March 1988), and also a small share after 2005; it also contributes to the accumulation of the effective inventory at the beginning and since 2004.

Figure 9 plot the same when $\gamma = 0.02$. Under the assumption of lower demand elasticity ($\gamma = 0.02$), the estimated cumulative effect of the persistent shock is similar as in the case of $\gamma = 0.25$. The model also estimates similar pattern for the cumulative effect of the expectation shock: the expectation shock contributes to a price spike in August 1991 (the outbreak of the Gulf War); it also contributes to the accumulation of the effective inventory in October 1990, and after 2004 except for July 2008 - March 2009 (the oil price peaked in June 2008). However, in terms of the magnitude of the effect, the expectation shock overall contributes more to of the price dynamics compared to in the case of $\gamma = 0.25$.

To illustrate and compare their relative contribution, Figure 10 rearranges the plotting and compare the historical decomposition under different γ's side by side. As discussed earlier, the overall patterns of the decomposed cumulative effects are similar, but there's difference in the magnitude. Overall, in both cases, the persistent shock is the largest contributor for the price dynamics, followed by the temporary shock, and the expectation shock; the temporary shock is the largest contributor to the effective inventory dynamics.

However, in the lower demand elasticity case ($\gamma = 0.02$), the model does attribute more of the price dynamics to the expectation shock. Table 4 provides more evidences. Table 4 presents the variance decomposition of forecast errors for the price and inventory k-month ahead under different γ's. The expectation shock contributes more than 150-fold when $\gamma = 0.02$. In terms of the variance decomposition, the expectation shock becomes the second largest contributor to the price dynamics after the persistent shock when $\gamma = 0.02$. On the other hand, the model attributes much less of the inventory dynamics to the expectation shock when $\gamma = 0.02$.

4.4 The Importance of the Price Elasticity of Demand

The different importance of the expectation shock estimated under different γ's is comparable to the literature finding. Even though both estimation results are from the same qualitative theoretical framework, the importance of the expectation shock is interpreted very differently.

Both Kilian and Murphy (2014) and Juvenal and Petrella (2014) analyze the contribution of expectations in the spot prices by the same qualitative sign restrictions using the price and inventory data. However, the two studies adopt different macroeconomic data, which reflect the demand side influence. In other words, the two studies implicitly derive different price demand elasticity of crude oil. Indeed, the price elasticity of demand can be inferred under the VAR framework, from the impact responses of production and of price to a supply shock. The two capture the movement along the demand curve when the supply curve is shifted by an exogenous supply shock. The ratio of the two is the price elasticity of demand. In specific, Kilian and Murphy (2014)'s impulse response functions imply a short-run price demand elasticity of -0.44. Though Juvenal and Petrella (2014) don't report the implied elasticity of demand, it can be inferred from the impulse response functions (Juvenal and Petrella (2014), Figure 2) that the short-run demand is less elastic (the elasticity is around -0.25) than Kilian and Murphy (2014)'s.

Earlier estimation results of the structural model shows that, the price elasticity of demand plays an important role in how the model attributes the observed price and inventory dynamics to the shocks. When the demand is inelastic ($\gamma = 0.02$), given the observed volatility and movements, the model can only allow for one shock with high persistence, and the expectation component is estimated to have more effect on the price rather than the inventory (compared to when $\gamma = 0.25$). Accordlingly the model attributes the more price dynamics to the expectation shock.

However, the importance of the elasticity is not implicit in the reduced-form analysis. The different macroeconomic data used implies different short-run price elasticity of demand, which changes the resulting interpretation of the shocks. Kilian and Murphy (2014) where the elasticity is higher, find little evidence for the expectations contributing to the price movements after 2000, while Juvenal and Petrella (2014) find more.

This result extends the discussion on the importance of the price elasticity of demand. This model illustrates that the less elastic the demand is assumed or implied by the data, the more of the price dynamics is attributed to the expectation component in order to reconcile with the observed data of price and inventory. This complements Hamilton (2009b)'s emphasis on the importance of elasticity in interpreting the role of speculation, where the the cases of perfectly price-inelastic demand versus slightly elastic demand are discussed. .

5 Conclusion

In this paper, I model market expectations explicitly in a structural model where the equilibrium prices and inventory decision are endogenously determined. Bringing the model to data, it's possible to analyze the contribution of expectations in the oil price dynamics. I consider the competitive inventory-holding decision of oil producers under the current and expected future market condition. The expectations of the future market condition is explicitly modeled as a shock that affects the relatively supply with a lag, following the news shock in the macroeconomic literature, in order to capture the forward-looking component in the price formation. In the model simulation, this expectation shock affects the price and inventory in a similar fashion as the speculative component that the literature has identified using reduced from models. Namely, in response to traditional concurrent shocks to relative supply (normalized to imply an increase in real spot price), spot price increases and inventory decreases. In response to expectation shock, spot price increases and inventory is accumulated. It's the different response profiles that enable identification of the different shocks from the data.

Under reasonable assumption of the price elasticity of demand (-0.25 and -0.02), the oil price movements have been mostly driven by the persistent shock, which characterizes a persisting constrained supply relative to the demand especially since 2000s. The constrained supply also result in the drawing down of the relative inventory since the end of 1999.

In addition, the short-run movements in the effective inventory are mostly contributed by the temporary shock, while the long-run trend in the relative inventory is driven by the persistent shock and the expectation shock together. The historical decomposition of the price and inventory dynamics even matches several historical events in the oil market.

In both sets of results, qualitatively the persistent shock drove inventory depletion after 1998 and the expectations drove inventory accumulation after 2004. This confirms an overall shift of the market expectations in 2000s.

More interestingly, in terms of the quantitative interpretation, the results show that the price elasticity of demand plays a key role. By comparing the estimation results under moderate (-0.25) and low (-0.02) demand elasticities, the model illustrates that the different elasticity setting can result in strikingly different interpretations. The lower the demand elasticity, the more the price dynamics is attributed to the expectation shock. It implies that in empirical studies, the different conclusions on the role of speculation could be a result of different implied demand elasticities due different demand data used.

This alternative explanation of the different results in the literature also illustrates the importance of the structural model. While the structural and the reduce-form models are comparable in many aspects, as discussed in the paper, the structural model has the advantage of explicitly modeling the economic decisions. The price elasticity of demand is implicitly included in the reduced-form models, and can be easily overlooked in the results interpretation. Resorting to the structural model reveals the key role of the price elasticity of demand in understanding the price behavior, and this is not just limited to the oil market.

While the current version of the model finds little evidence for the expectations driving up the price in the 2000s (especially under the assumption of moderate demand elasticity), this could have to do with how the expectation is modeled. The expectation shock is a shock to the relative supply with one-period lag, and thus captures expectations of the level of future relative supply. However, the speculative incentives also include increased uncertainty about future market condition, which can be modeled as a mean-reserving volatility increase of the relative supply. This would affect prices and inventory decision without changing future relative supply, which cannot be captured by the current expectation shock. As Kilian and Murphy (2014) point out, "news about the level of future oil supplies and the level of future demand for crude oil are but one example of shocks to expectations in the global market for crude oil." Such expectation shock can be explored in the future work.

A Solving the Model

To solve the detrended model in Section 2.4, first I find its steady state and log-linearize the model around the steady state, second I solved the log-linearized linear system using Blanchard and Kahn (1980) and write the model in a state-space form.

First, I write out the steady state of the model in Section 2.4 (the steady state values are in bold; for example $n_t = n_{t+1} = \boldsymbol{n}$ in steady state):

$$\boldsymbol{P} = c[(\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - n) * \boldsymbol{q^s}]^{-\frac{1}{\gamma}} \tag{15}$$

$$1 = \beta - [\alpha(\frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}})^{-\phi} + \delta] \tag{16}$$

$$\log \boldsymbol{\mu^s} = \bar{\mu} \tag{17}$$

$$\log \boldsymbol{q^s} = 0 \tag{18}$$

$$\boldsymbol{y^{\tau}} = 0 \tag{19}$$

$$\boldsymbol{y^{c}} = 0 \tag{20}$$

$$\boldsymbol{n^{\tau}} = 0 \tag{21}$$

Then I log-linearize the model in Section 2.4 around the steady state.

Define $\hat{P}_t = (P_t - \boldsymbol{P})/\boldsymbol{P}$, $\hat{n}_t = (n_t - \boldsymbol{n})/\boldsymbol{n}$, $\hat{\mu}^s_t = (\mu^s_t - \boldsymbol{\mu^s})/\boldsymbol{\mu^s}$, $\hat{q}^s_t = (q^s_t - \boldsymbol{q^s})/\boldsymbol{q^s}$ for all t, the original model in Section 2.4 can be written as terms of the deviation from the steady state:

$$\hat{P}_t = -\frac{1}{\gamma}[pn_0\hat{n}_t - pn_1\hat{n}_{t+1} - pu\hat{\mu}^s_t + py\hat{q}^s_t] \tag{22}$$

where

$$pn_0 = \frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{23}$$

$$pn_1 = \frac{\boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{24}$$

$$pu = \frac{\boldsymbol{n}/\boldsymbol{\mu^s}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} \tag{25}$$

$$py = 1 \tag{26}$$

$$\hat{P}_t = \beta E_t[\hat{P}_{t+1}] - \frac{\boldsymbol{MIC}}{\boldsymbol{P}}E_t[M\hat{I}C_{t+1}] \tag{27}$$

$$M\hat{I}C_{t+1} = \hat{P}_t + micn_0\hat{n}_t + micn_1\hat{n}_{t+1} + micn_2\hat{n}_{t+2} + micu_0\hat{\mu}^s_t + micu_1\hat{\mu}^s_{t+1} \tag{28}$$

where

$$micn_0 = -\frac{1}{\beta - 1} * \Theta' * \boldsymbol{\mu^s} \tag{29}$$

$$micn_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{1 - \boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} + (1 + \beta) * \Theta' * \boldsymbol{\mu^s}] \tag{30}$$

$$micn_2 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{\boldsymbol{n}}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} - \beta * \Theta' * \boldsymbol{\mu^s}] \tag{31}$$

$$micu_0 = \frac{1}{\beta - 1} * \Theta' * \boldsymbol{\mu^s} \tag{32}$$

$$micu_1 = \frac{1}{\beta - 1}[\phi(1 - \beta + \delta)\frac{\boldsymbol{n} - 1}{\boldsymbol{n}/\boldsymbol{\mu^s} + 1 - \boldsymbol{n}} - \beta * \Theta' * \boldsymbol{\mu^s}] \tag{33}$$

Following Blanchard and Kahn (1980), the log-linearized model's variables are grouped as state variables X_t, costate variables Y_t and exogenous shock variables e_t, where $X_t' = \begin{bmatrix} \hat{n}_t & \hat{n}_{t+1} \end{bmatrix}$, $Y_t = \begin{bmatrix} \hat{P}_t \end{bmatrix}$, $e_t' = \begin{bmatrix} \hat{\mu}_t^s & y_t^\tau & y_t^c & n_t^\tau \end{bmatrix}$. The above model can be solved for the state-space form (or more specifically, to solve for F, Z, U, H and R in the state-space form below from Equation (22) - (33)).

The resulting state-space model is in the format below:

State equation:

$$\begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} = F \begin{bmatrix} \hat{n}_{t-1} \\ e_{t-1} \end{bmatrix} + Z * v_t \qquad v_t \sim N(0, U) \tag{34}$$

where $v_t' = \begin{bmatrix} \epsilon_t^{\mu^s} & \epsilon_t^{y_\tau} & \epsilon_t^{y_c} & \epsilon_t^{n_\tau} \end{bmatrix}$, $Z = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $U = \begin{bmatrix} \sigma^2_{\mu^s} & 0 & 0 & 0 \\ 0 & \sigma^2_{y_\tau} & 0 & 0 \\ 0 & 0 & \sigma^2_{y_c} & 0 \\ 0 & 0 & 0 & \sigma^2_{n_\tau} \end{bmatrix}$.

Observation equation:

$$\hat{P}_t = H \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + u_{1t} \qquad u_t \sim N(0, R_1) \tag{35}$$

where u_{1t} is the measurement error for the spot price, and its variance is a small positive number (in the estimation it's set to be 1/100000).

A.1 Additional Observables

In addition to the spot market, crude oil futures contracts are also actively traded. If 1-month futures price approximates of the expected 1-month ahead spot price, the futures price can serve as another observed variable.

The state space model implies the following for the 1-month ahead expected price:

$$E_t\hat{P}_{t+1} = H \begin{bmatrix} E_t\hat{n}_{t+1} \\ E_t e_{t+1} \end{bmatrix} \tag{36}$$

$$= H * F * \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} \tag{37}$$

This gives rise to the second observation equation:

$$\hat{F}_{t,1} = H * F * \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + u_{2t} \qquad u_t \sim N(0, R_2) \tag{38}$$

where $F_{t,1}$ is the 1-month futures price quoted at t and u_{2t} is the measurement error for the futures price, and its variance is a small positive number (in the estimation it's set to be 1/100000).

A.2 Observable State Variables

One advantage of the model is that two of the state variables are actually observed. Both the effective inventory $\hat{n}_{t+1}$ and the world supply growth rate $\hat{\mu}_t^s$ are available. This provides two additional observation equations in the state-space form:

$$\begin{bmatrix} \hat{n}_t \\ \hat{\mu}_t^s \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ e_t \end{bmatrix} + \begin{bmatrix} \epsilon_t^{\hat{n}} \\ 0 \end{bmatrix} \qquad \epsilon_t^n \sim N(0, \sigma_{\hat{n}}^2) \tag{39}$$

where ϵ_t^n is the measurement error for the effective inventory. This allows for correcting possible data inaccuracy due to using the OECD effective inventory as the proxy of world inventory. On the other hand, the world supply growth rate $\hat{\mu_t^s}$ already contains a shock in the state equation (see Equation 34).

In order to remove the seasonality in the inventory data, monthly dummies are included in the inventory observation equation, so that in the estimation:

$$\hat{n}_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{n}_t \\ \\ e_t \end{bmatrix} + d_i + \epsilon_t^{\hat{n}} \quad \epsilon_t^n \sim N(0, \sigma_{\hat{n}}^2) \tag{40}$$

where d_i is the dummy variable for month i, $d_i = 0$ if $i = March$; $d_i \neq 0$ otherwise.

A.3 Equations for the Estimation

To summarize, the equations in the estimation are Equations 34 35 38 40 and the second row (for $\hat{\mu}_t^s$) of Equation 39.

B Estimation of the State Space Model

Given a starting set of parameters, with the state equation 34, the observation equations 35 38 40 and the second row (for $\hat{\mu}_t^s$) of Equation 39, and the observed data, I use the Kalman filter to produce the estimates of the state variables, as well as the joint likelihood under this set of parameter. The maximum likelihood estimation of the model involves finding the parameters to maximize the joint likelihood. Once the parameters are estimated, the estimates of the state variables are also produced, and smoothed by Kalman smoother. The state variables and the decomposition results discussed in the paper are all based on smoothed state variables.

For the results discussion, the smoothed state variables are not plotted. Rather the historical decomposition and variance decomposition are provided for better illustration. The figures of the state variables can be provided on request.

To compute the historical decomposition of the price and inventory, aside from the shock of interest, all other shocks are set to be zeros over the whole sample period. The effective inventory in the first period is also set to be zero. The hypothetical price and inventory over time is calculated iteratively from the time path of the shock of interest, using the estimated state space model. Thus the historical decomposition of the inventory always starts from zero in figures.

Table 1: **Model Parameterization**

Parameters	Value	Description
β	0.997	monthly depreciation rate
γ	0.25	price elasticity of demand
ϕ	1.42	parameter in MIC
Θ'	0.2	marginal cost of inventory change
δ	0.89	marginal physical storage cost
ρ^{τ}	0.9	AR coef of persistent shock
ρ^{c}	0.1	AR coef of temporary shock
ρ^{n_τ}	0.9	AR coef of expectation shock
σ_{y_τ}	1	s.d. of persistent shock
σ_{y_c}	1	s.d. of temporary shock
σ_{n_τ}	1	s.d. of expectation shock
σ_{μ^s}	1	s.d. of growth rate shock
$\sigma_{\hat{n}}$	1	s.d. of inventory measurement error[a]

[a]In the observation equation, although the observed effective inventory is mapped 1 to 1 directly from the state variable effective inventory, I allow for measurement errors in the observed values.

Table 2: **Estimated Model for Crude Oil Market**

Parameters	$\gamma = 0.25$		$\gamma = 0.02$		Description
	Point Estimate	(Standard Error)	Point Estimate	(Standard Error)	
log likelihood	4628		4635		
β (set)	0.997		0.997		monthly depreciation rate
γ (set)	0.25		0.02		price elasticity of demand for crude oil
ϕ (set)	1.42		1.42		parameter in net marginal convenience yield
Θ'	0.0151^{***}	(0.0004)	0.0018^{***}	(0.0002)	
δ	0.0025^{***}	(0.0001)	0.0021^{***}	(0.0001)	marginal physical storage cost
ρ^{τ}	0.9993^{***}	(0.0000)	0.9998^{***}	(0.0000)	AR coefficient of persistent shock
ρ^{c}	0.0451^{***}	(0.0035)	0.0279^{***}	(0.0011)	AR coefficient of temporary shock
ρ^{n_τ}	0.9991^{***}	(0.0000)	0.0000^{***}	(0.0000)	AR coefficient of expectation shock
σ_{y_τ}	0.0197^{***}	(0.0001)	0.0010^{***}	(0.0002)	s.d. of persistent shock
σ_{y_c}	0.0092^{***}	(0.0003)	0.0088^{***}	(0.0015)	s.d. of temporary shock
σ_{n_τ}	0.0000^{***}	(0.0000)	0.0003^{***}	(0.0000)	s.d. of expectation shock
σ_{μ^s} (set)	0.0105		0.0105		s.d. of growth rate shock
$\sigma_{\hat{n}}$	0.0000^{***}	(0.0000)	0.0000^{***}	(0.0000)	s.d. of inventory measurement error[a]

Note: (i) Standard errors of the estimates are simulated and reported in parentheses; (ii) *, ** and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

Table 3: **Estimated Model for Crude Oil Market - continued**

Parameters	$\gamma = 0.25$		$\gamma = 0.02$		Description
	Point Estimate	(Standard Error)	Point Estimate	(Standard Error)	
log likelihood	4628		4635		
Jan.	-0.0377^{***}	(0.0052)	-0.0383	(0.0329)	monthly seasonality dummy
Feb.(set)	-0.0105^{***}	(0.0036)	-0.0109	(0.1042)	monthly seasonality dummy
Mar.	0		0		monthly seasonality dummy
Apr.	0.0300^{***}	(0.0037)	0.0305	(0.0308)	monthly seasonality dummy
May.	0.0419^{***}	(0.0050)	0.0429	(0.0307)	monthly seasonality dummy
Jun.	0.0337^{***}	(0.0060)	0.0348	(0.0309)	monthly seasonality dummy
Jul.	0.0112^{*}	(0.0063)	0.0115	(0.0312)	monthly seasonality dummy
Aug.	-0.0041	(0.0066)	-0.0040	(0.0646)	monthly seasonality dummy
Sep.	-0.0129^{*}	(0.0067)	-0.0132	(0.0440)	monthly seasonality dummy
Oct.	-0.0333^{***}	(0.0064)	-0.0339	(0.0317)	monthly seasonality dummy
Nov.	-0.0068	(0.0063)	-0.0073	(0.0353)	monthly seasonality dummy
Dec.	-0.0121^{**}	(0.0057)	-0.0130	(0.0480)	monthly seasonality dummy

Note: (i) Simulated standard errors of the estimates are in parentheses (20000 simulations); (ii) *, ** and ***denote that the point estimate is significant at the 90%, 95% and 99% confidence levels, respectively.

Table 4: **The Variance Decomposition k-month Ahead under Different γ's**

Forecast error in	Innovation in	γ	$k=1$	$k=3$	$k=6$	$k=12$	$k=24$
P_t	y^τ	$\gamma=0.25$	0.9967	0.9974	0.9976	0.9978	0.9975
		$\gamma=0.02$	0.9501	0.9515	0.9526	0.9546	0.9573
	y^c	$\gamma=0.25$	0.0013	0.0007	0.0004	0.0002	0.0000
		$\gamma=0.02$	0.0109	0.0096	0.0086	0.0068	0.0043
	n^τ	$\gamma=0.25$	0.0002	0.0002	0.0002	0.0003	0.0008
		$\gamma=0.02$	0.0350	0.0351	0.0352	0.0354	0.0357
n_{t+1}	y^τ	$\gamma=0.25$	0.0016	0.0073	0.0261	0.1042	0.2686
		$\gamma=0.02$	0.0000	0.0000	0.0001	0.0004	0.0017
	y^c	$\gamma=0.25$	0.8223	0.8083	0.7600	0.5606	0.1473
		$\gamma=0.02$	0.8123	0.8126	0.8126	0.8124	0.8115
	n^τ	$\gamma=0.25$	0.0034	0.0155	0.0551	0.2181	0.5533
		$\gamma=0.02$	0.0009	0.0008	0.0008	0.0007	0.0005

Note: (i) P_t: the spot price in period t; n_{t+1}: the effective inventory determined in period t for the beginning of period $t+1$; (iI) y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock.

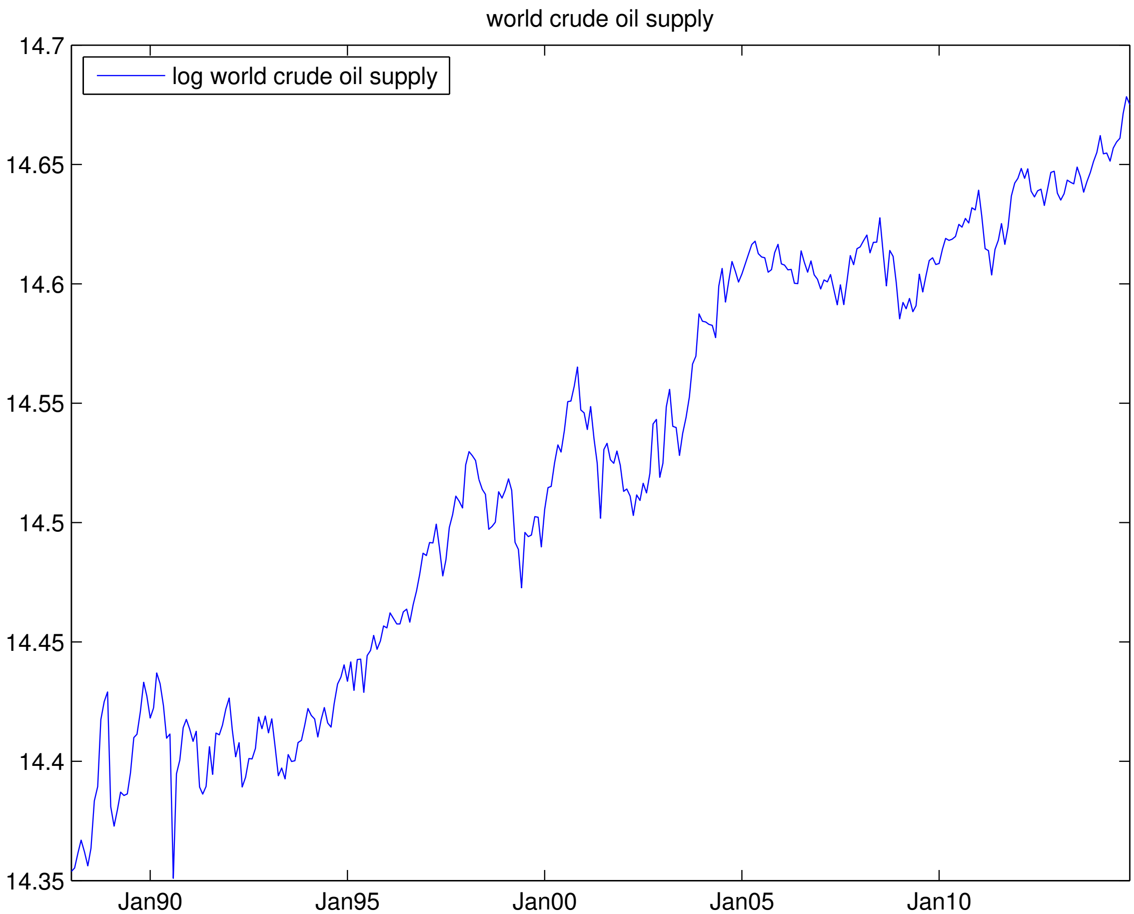


Figure 1: World Supply of Crude Oil

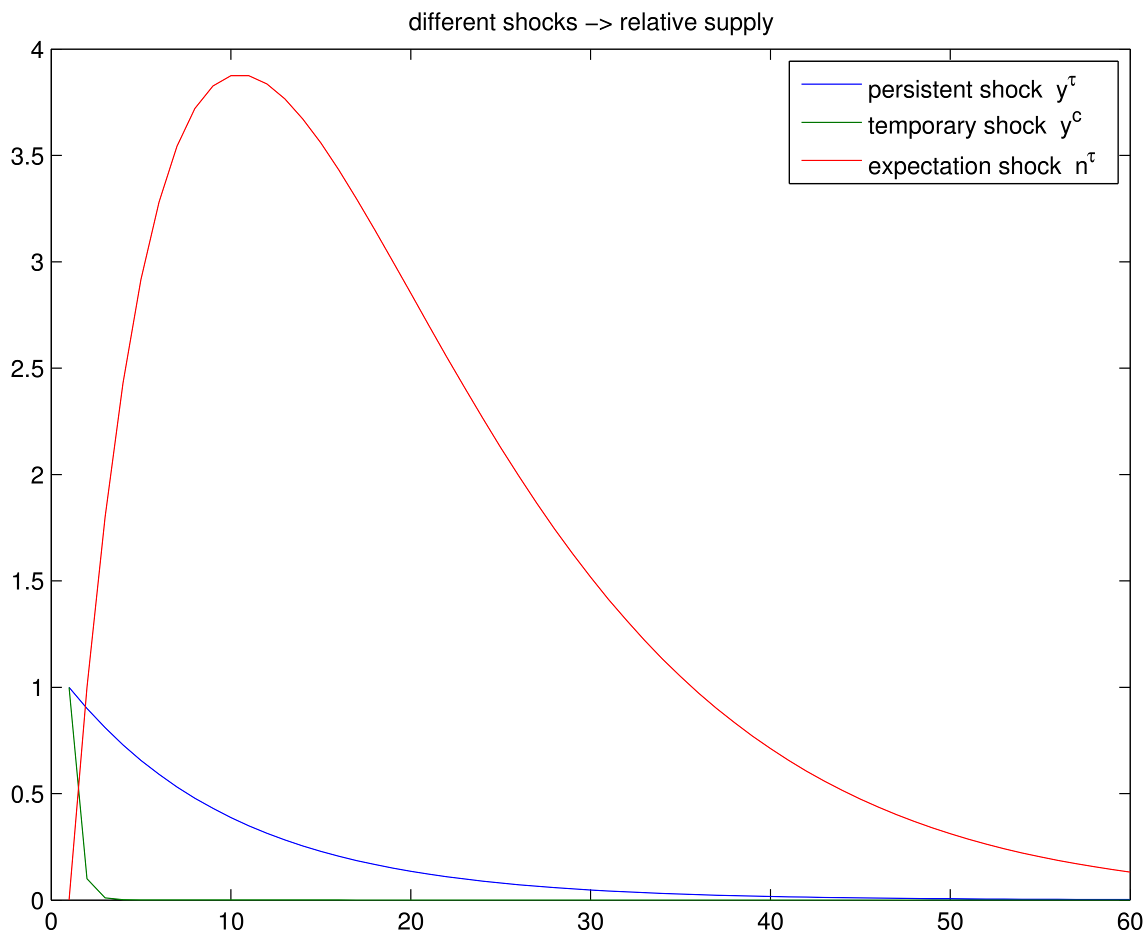


Figure 2: Effect of the Shocks on Relative Supply under Arbitrary Parameterization

Note: All shocks have been normalized to cause an increase in the relative supply.

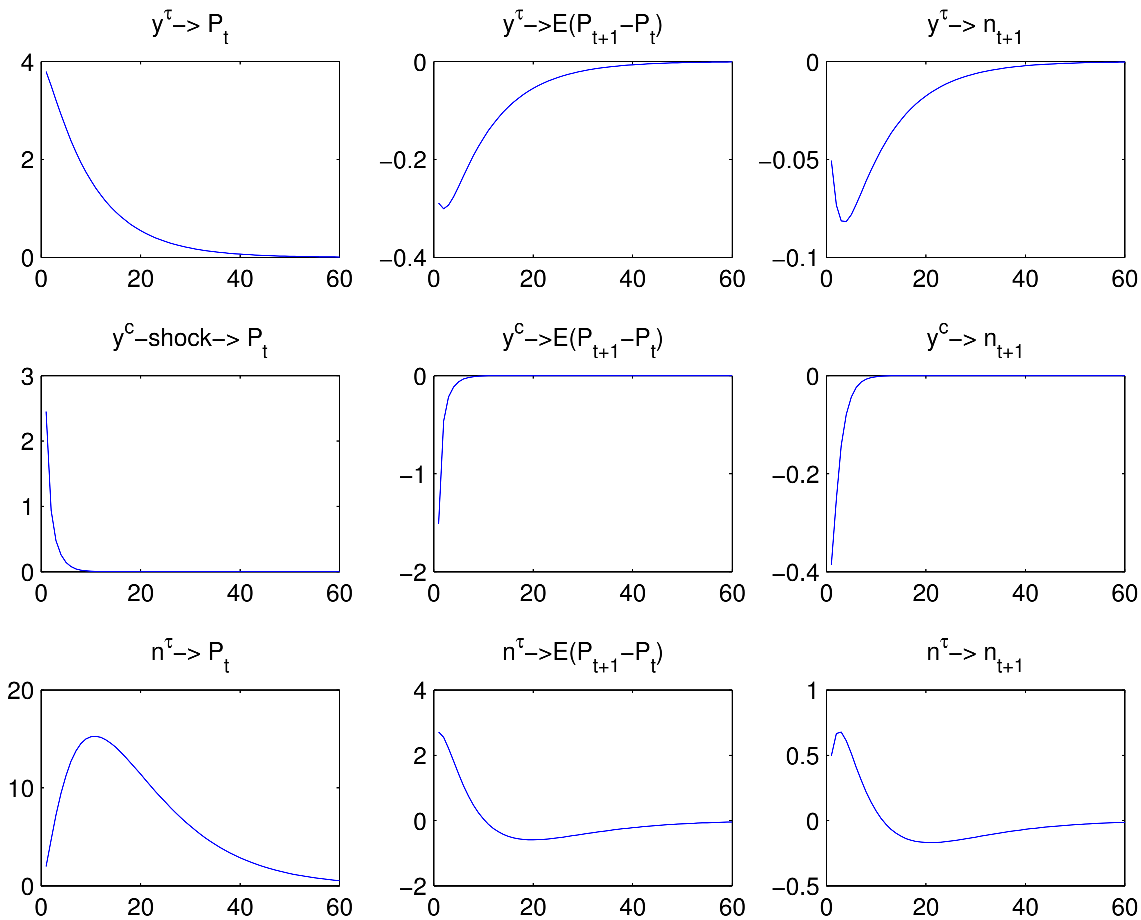


Figure 3: Impulse Response Functions under Arbitrary Parameterization

Note: 1. y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.

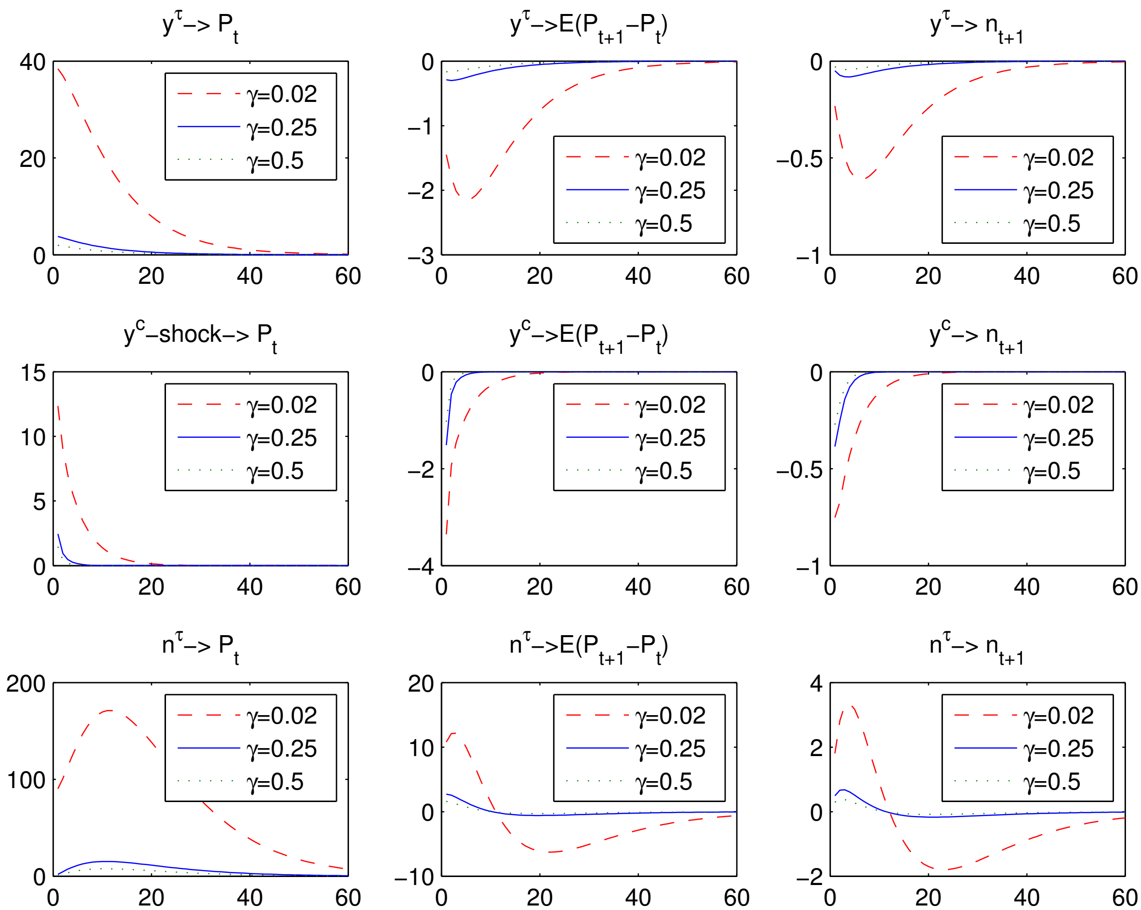
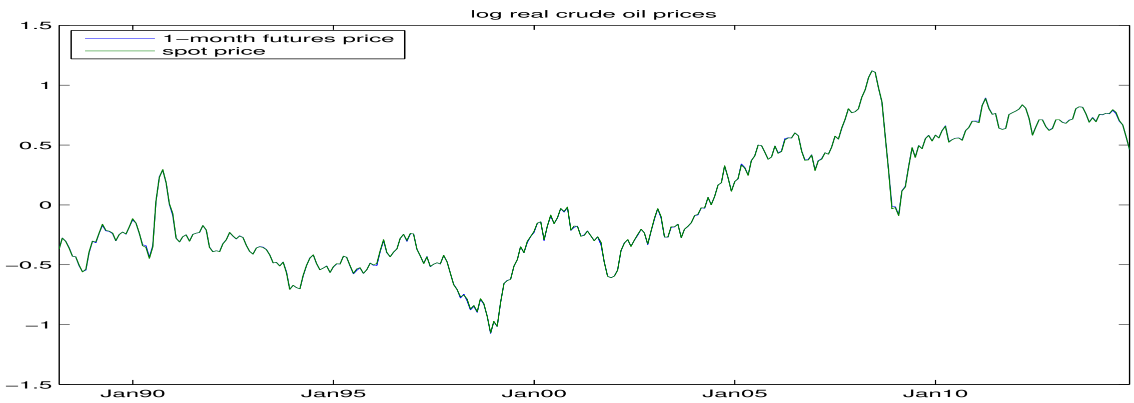
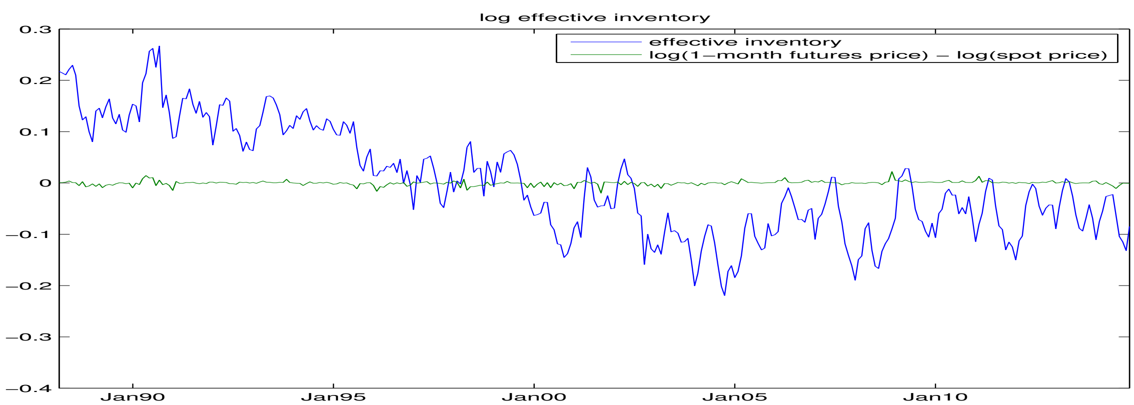


Figure 4: Impulse Response Functions under Arbitrary Parameterization with different γ's

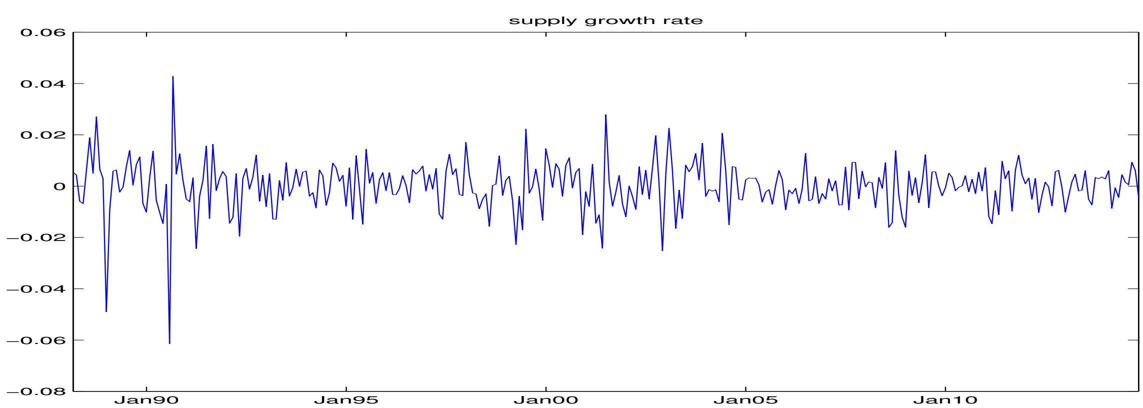
Note: 1. y^{τ}: persistent shock; y^{c}: temporary shock; n^{τ}: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.



(a) WTI Prices

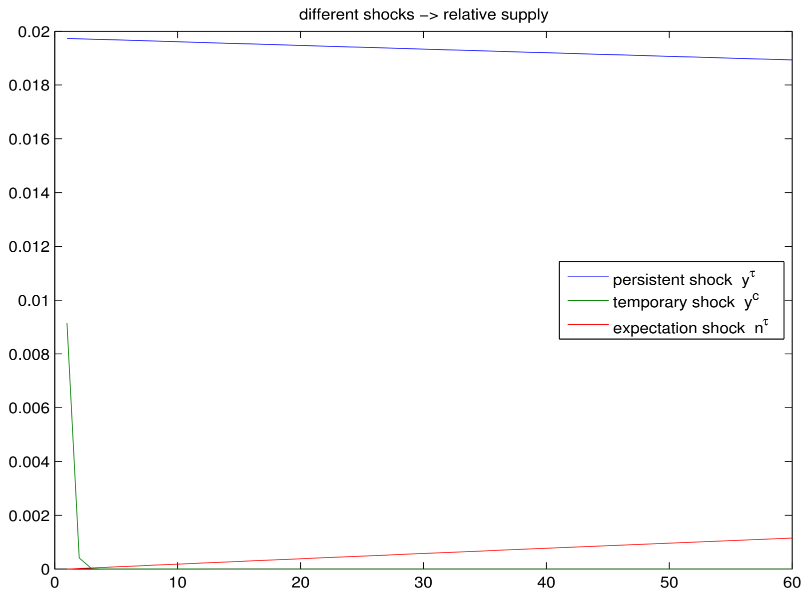


(b) Effective Inventory

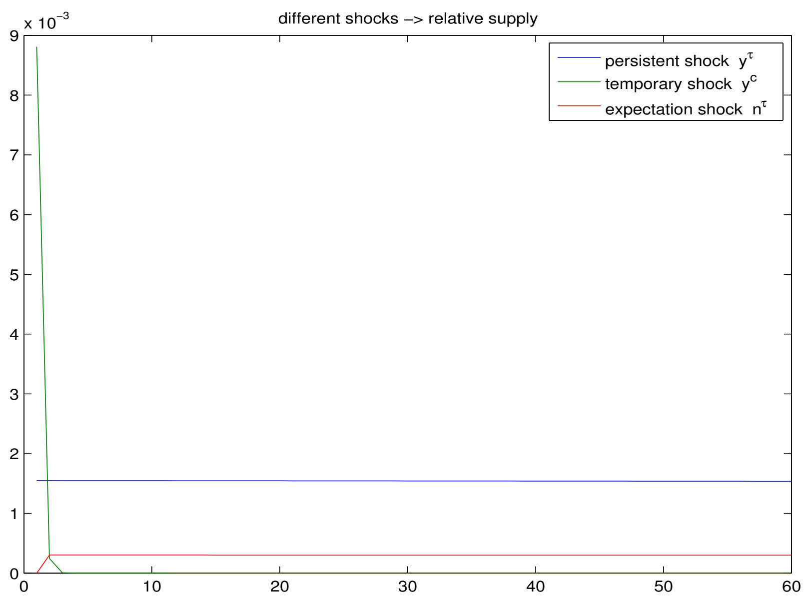


(c) Supply Growth Rate

Figure 5: Data Overview



(a) $\gamma = 0.25$



(b) $\gamma = 0.02$

Figure 6: Estimated Effect of the Shocks on Relative Supply

Note: All shocks have been normalized to cause an increase in the relative supply.

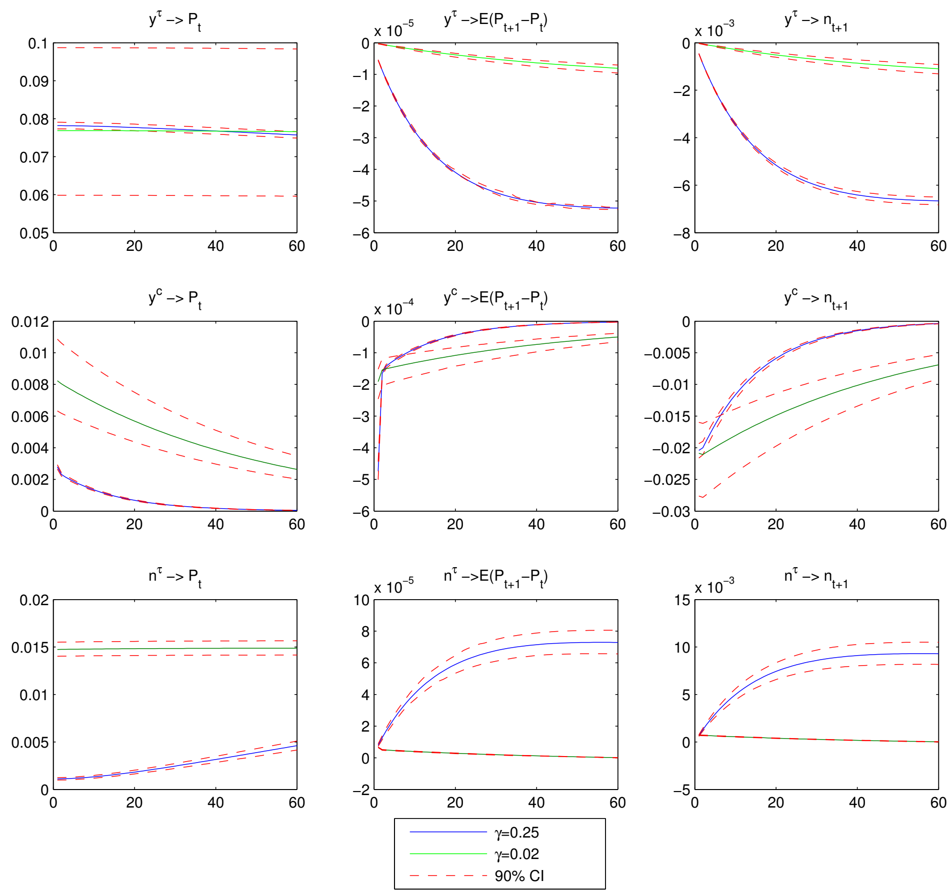


Figure 7: Estimated Impulse Response Functions

Note: 1. y^τ: persistent shock; y^c: temporary shock; n^τ: expectation shock; 2. All shocks have been normalized to cause an increase in the real spot price of oil.

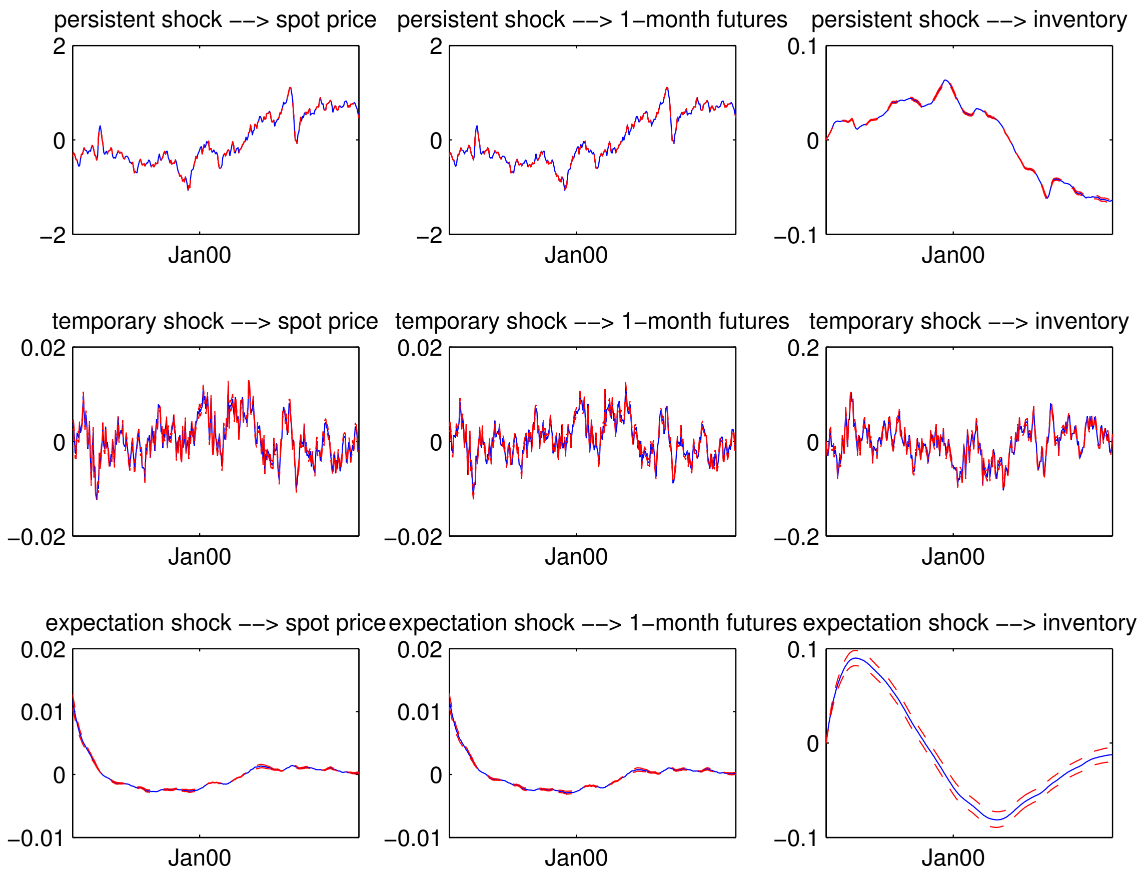


Figure 8: Cumulative Effect of Shocks on the Prices and Effective Inventory with 90% CI: $\gamma = 0.25$

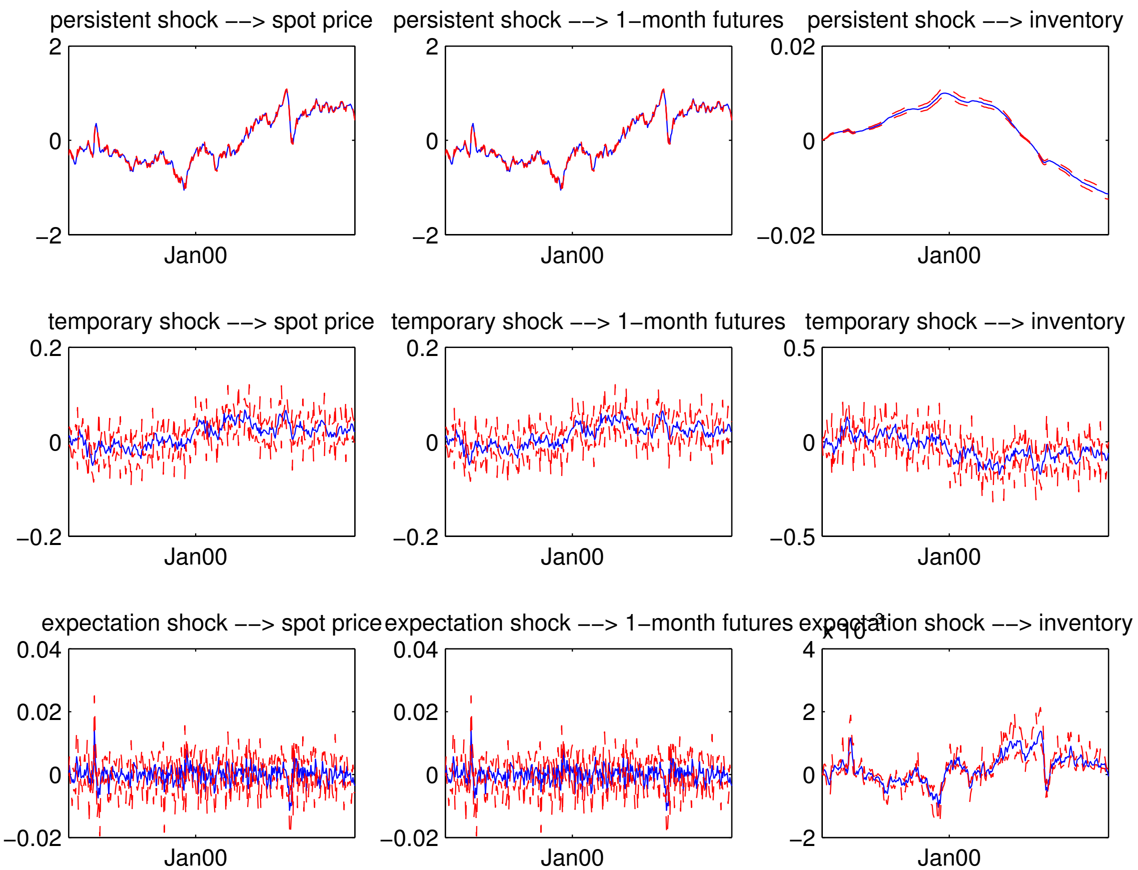
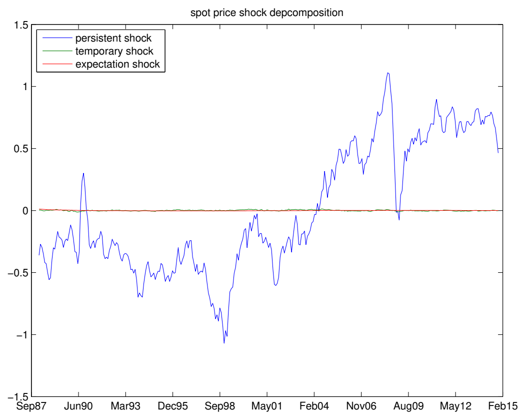
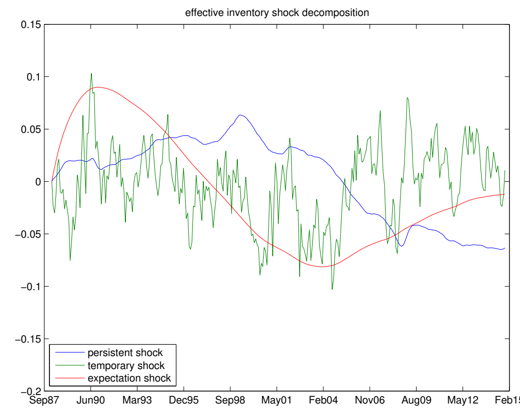


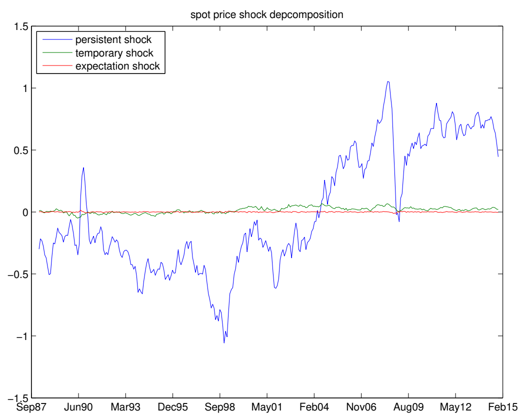
Figure 9: Cumulative Effect of Shocks on the Prices and Effective Inventory with 90% CI: $\gamma = 0.02$



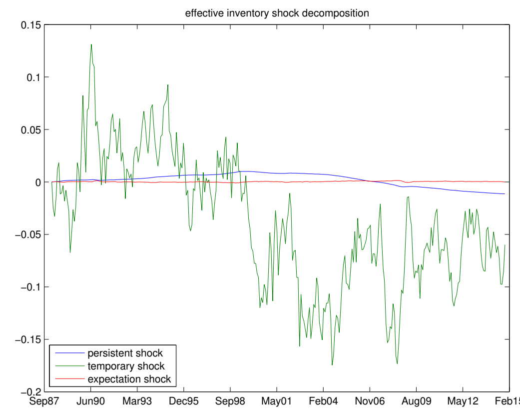
(a) $\gamma = 0.25$: Spot Price Decomposition



(b) $\gamma = 0.25$: Effective Inventory Decomposition



(c) $\gamma = 0.02$: Spot Price Decomposition



(d) $\gamma = 0.02$: Effective Inventory Decomposition

Figure 10: Cumulative Effect of Shocks to Price and Inventory

Note: For illustration purpose, the CI's from Figure 8 and Figure 9 are not included in the rearranged plottings.

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